Information Theoretically Secure Hypothesis Test for Temporally Unstructured Quantum Computation

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The efficient certification of classically intractable quantum devices has been a central research question for some time. However, to observe a "quantum advantage", it is believed that one does not need to build a large scale universal quantum computer; a task which has proven extremely challenging. Intermediate quantum models that are easier to implement, but which also exhibit this quantum advantage over classical computers, have been proposed. In this work, we present a certification technique for such a sub-universal quantum server which only performs commuting gates and requires very limited quantum memory. By allowing a verifying client to manipulate single qubits, we exploit properties of measurement based blind quantum computing to give them the tools to prove the "quantum superiority" of the server.

1 Introduction

Quantum computers are believed to be able to efficiently simulate some quantum systems [15, 19] while other protocols demonstrating their power include Shor's algorithm for prime factorisation [36], Grover's algorithm for unstructured search [20], and the BB84 [5] and Ekert91 [14] protocols for public key exchange. That said, it may be some time before a large scale universal quantum computer capable of demonstrating the computational power of these protocols is built. In the meantime several intermediate, non-universal models of quantum computation, which are still believed to not be classically simulatable, may prove easier to implement. Examples of such models include the one clean qubit model [25, 28], the boson sampling model [1, 18, 30] and the Ising model [17, 29]. The *Instantaneous Quantum Poly-time* (IQP) machine [35] is another such non-universal model with significant practical advantages [2, 6]. IQP uses only commuting gates but is believed to remain hard to classically simulate [6, 7, 9] even in a noisy environment [8]. Hence, providing evidence that a machine can perform hard IQP computations would be a proof of quantum superiority before a universal quantum computer has been experimentally realised.

In [35], the authors present a *hypothesis test* which can be passed only by devices capable of performing hard IQP computations. In order to accommodate a purely classical client, computational assumptions (conjecturing the hardness of finding hidden sub-matroids) are required in order to prove quantum superiority. In the present work, by endowing the Client with the ability to perform simple qubit manipulations similar to that used in Quantum Key Distribution schemes [5], we develop an information-theoretically secure hypothesis test for IQP.

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The remainder of the paper proceeds as follows. In Section 2, we formally introduce the IQP machine and provide an implementation in Measurement Based Quantum Computing (MBQC) [33, 34] which is more suitable for proving security in our framework than previous ones [35, 22]. In Section 3 we use tools from blind quantum computing [10, 16] to derive a delegated protocol for IQP computations which keeps the details of the computation hidden from the device performing it. We prove information-theoretic security of that scheme in a composable framework. In Section 4 we develop an information theoretically secure hypothesis test that a limited quantum Client can run to certify the quantum superiority of an untrusted Server. Proofs of the lemmas and theorems found in this work are given in full in [27].

2 Preliminaries

2.1 X-programs

The IQP machine introduced in [35] is defined by its capacity to implement X-programs.

Definition 2.1. *An* X-program *consists of a Hamiltonian comprised of a sum of products of X operators on different qubits, and $\theta \in [0, 2\pi]$ describing the time for which it is applied. The h-th term of the sum has a corresponding vector $\mathbf{q}_h$, called a* program element*, which defines on which of the n_p input qubits, the product of X operators, which constitute that term, acts. The vector $\mathbf{q}_h$ has 1 in the j-th position when X is applied on the j-th qubit.*

As such, we can describe the X-program using θ and a poly-size list of n_a vectors $\mathbf{q}_h \in \{0,1\}^{n_p}$ or, if we consider the matrix $\mathbf{Q}$ which has as rows the program elements $\mathbf{q}_h$, $h = 1, \ldots, n_a$, simply by the pair $(\mathbf{Q}, \theta) \in \{0,1\}^{n_a \times n_p} \times [0, 2\pi]$.

Applying the X-program discussed above to the computational basis state $|0^{n_p}\rangle$ and measuring the result in the computational basis allows us to see an X-program as a quantum circuit with input $|0^{n_p}\rangle$, comprised of gates diagonal in the Pauli-X basis, and classical output. Using the random variable X to represent the distribution of output samples, the probability distribution of outcomes $\widetilde{x} \in \{0,1\}^{n_p}$ is:

$$\mathbb{P}(X = \widetilde{x}) = \left| \langle \widetilde{x} | \exp \left(\sum_{h=1}^{n_a} i\theta \bigotimes_{j:\mathbf{Q}_{hj}=1} X_j \right) |0^{n_p}\rangle \right|^2 \tag{1}$$

Definition 2.2. *Given some X-program, an* IQP machine *is any computational method capable of efficiently returning a sample $\widetilde{x} \in \{0,1\}^{n_p}$ from the probability distribution* (1)*.*

2.2 IQP In MBQC

A common framework for studying quantum computation is the MBQC model [33], where a quantum operation is expressed by a set of measurement angles on an entangled state described by a graph. This entangled state is built by applying a controlled-Z operation between qubits when there is an edge in the corresponding graph. The probabilistic nature of the measurements of qubits in this state introduces some randomness, but this can be corrected for by adjusting the angle of measurement of subsequent qubits depending on the outcomes of the already performed measurements. The entangling, measuring and correcting operations on a set of qubits are usually referred to as a *measurement pattern* [11, 12].

In this work, we will deal with a specific type of graphs, given in the following definition:

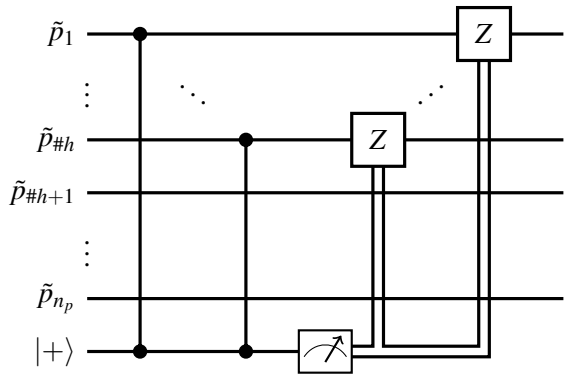
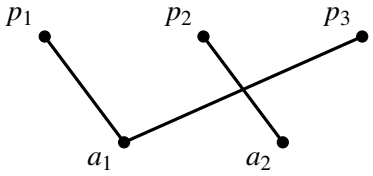


Figure 1: The circuit implementing one term in the sum of equation (1). The input qubits $\{p_j\}_{j=1}^{n_p}$ are rearranged so that if $\#h$ is the Hamming weight of row h of matrix $\mathbf{Q}$, then for $k = 1,\ldots,\#h$ each $\tilde{p}_k$ corresponds to one p_j such that $\mathbf{Q}_{hj} = 1$ and for $k = \#h+1,\ldots,n_p$ they correspond to the ones such that $\mathbf{Q}_{hj} = 0$. The ancillary qubit measurement is in the basis $\{|0_\theta\rangle, |1_\theta\rangle\}$ defined in expression (2).



$$\mathbf{Q} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Figure 2: An example of an IQP graph described by matrix $\mathbf{Q}$. Here, $n_p = 3$ and $n_a = 2$ while the partition used is $P = [p_1, p_2, p_3]$ and $A = [a_1, a_2]$.

Definition 2.3. *An* undirected bipartite graph*, which we refer to as an* IQP graph*, consists of a bipartition of vertices into two sets P and A of cardinality n_p and n_a respectively.*

We may represent such a graph by $\mathbf{Q} \in \{0,1\}^{n_a \times n_p}$. An edge exists in the graph when $\mathbf{Q}_{hj} = 1$, for $h = 1,\ldots,n_a$ and $j = 1,\ldots,n_p$. We call the set P primary vertices *and the set A* ancillary vertices*.*

A result which is vital to the remainder of this paper, is the following:

Lemma 2.1. *A measurement pattern can always be designed to simulate an X-program efficiently.*

We can prove that the distribution of (1) may be achieved by initialising n_p *primary qubits* in the states $|p_j\rangle = |+\rangle$, n_a *ancillary qubits* in the states $|a_h\rangle = |+\rangle$, applying Controlled-Z operations between qubits when there is an edge in the bipartite graph described by the X-program matrix $\mathbf{Q}$ and measuring the resulting state. We form this proof by demonstrating that producing the distribution in equation (1) can be achieved by inputting the state $|+^{n_p}\rangle$ into a circuit made from composing circuits like the one in Figure 1 (one for each term of the sum in equation (1)) and measuring the result in the Hadamard basis. We then argue that all measurements may be delayed to the end of the circuit build from composing those of Figure 1. The ancillary measurement basis is:

$$\{|0_\theta\rangle, |1_\theta\rangle\} = \left\{ \frac{1}{\sqrt{2}} \left(e^{-i\theta} |+\rangle + e^{i\theta} |-\rangle \right), \frac{1}{\sqrt{2}} \left(e^{-i\theta} |+\rangle - e^{i\theta} |-\rangle \right) \right\} \tag{2}$$

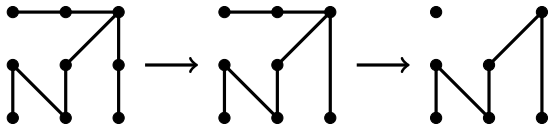


Figure 3: An example of a sequence of one bridge and one break operation.

3 Blind Delegated IQP Computation

We now move to build a method for blindly performing an IQP computation in a delegated setting. We consider a Client with limited quantum power delegating an IQP computation to a powerful Server. The novel method that we use in this work is to keep the X-program secret by concealing the quantum state used. Intuitively, this is done by the Client asking the Server to produce a quite general quantum state and then move from that one to the one that is required for the computation. If this is done in a blind way then the Server only has knowledge of the general starting state from which any number of other quantum states may have been built. Hence, there are three key problems to be addressed:

1. How to transform one state to another.
2. Which general quantum state to transform into the one used for IQP computations.
3. How to do so secretly in a delegated setting.

3.1 Break and Bridge

The break and bridge operations [16, 21] on a graph $\widetilde{G} = (\widetilde{V}, \widetilde{E})$, with vertex set $\widetilde{V}$ and edge set $\widetilde{E}$ describe the operations necessary to solve problem 1.

Definition 3.1. *The* break *operator acts on a vertex $v \in \widetilde{V}$ of degree 2 in a graph $\widetilde{G}$. It removes v from $\widetilde{V}$ and also removes any edges connected to v from $\widetilde{E}$.*

The bridge *operator also acts on $v \in \widetilde{V}$ of degree 2 in a graph $\widetilde{G}$. It removes v from $\widetilde{V}$, removes edges connected to v from $\widetilde{E}$ and adds an edge between the neighbours of v.*

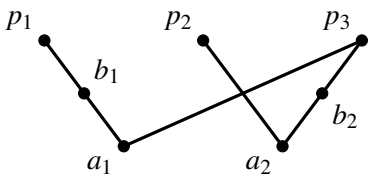
Figure 3 gives an example of multiple applications of the bridge and break operators, while *extended IQP graphs* describe the states which solve problem 2.

Definition 3.2. *An* extended IQP graph *is represented by $\widetilde{\mathbf{Q}} \in \{-1, 0, 1\}^{n_a \times n_p}$. The vertex set contains $A = \{a_1, ..., a_{n_a}\}$ and $P = \{p_1, ..., p_{n_p}\}$ while $\widetilde{\mathbf{Q}}_{hj} = 0$ and $\widetilde{\mathbf{Q}}_{hj} = 1$ has the same implications, regarding the connections between these vertices, as in IQP graphs.*

We interpret $\widetilde{\mathbf{Q}}_{hj} = -1$ as the existence of an intermediary vertex b_k between vertices p_j and a_h, and denote with n_b the number of -1s in $\widetilde{\mathbf{Q}}$. The vertex set includes the bridge and break vertices *$B = \{b_1, ..., b_{n_b}\}$ and the edge set includes edges between b_k and a_h as well as between b_k and p_j when $\widetilde{\mathbf{Q}}_{hj} = -1$. We define a surjective function g for which $g(h, j) = k$ when b_k is the intermediate vertex connected to a_h and p_j.*

An *extended IQP graph* $\widetilde{\mathbf{Q}}$ can be built from an IQP graph $\mathbf{Q}$ by replacing any number of the entries of $\mathbf{Q}$ with -1. Throughout the remainder of this work we will use tilde notation to represent an extended IQP graph $\widetilde{\mathbf{Q}}$ build from an IQP graph $\mathbf{Q}$ in this way. Figure 4 displays an example of an extended IQP graph. By applying a bridge operator to b_1 and a break operation to b_2 in $\widetilde{\mathbf{Q}}$ of Figure 4 we arrive at $\mathbf{Q}$ of Figure 2. It is in this sense that an extended IQP graph is 'more general' that an IQP graph.

To solve our three problems we must translate these graph theoretic ideas into operations on quantum states. The following definition allows us to use graphs defined above to describe entanglement patterns.



$$\widetilde{\mathbf{Q}} = \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

Figure 4: An extended IQP graph described by $\widetilde{\mathbf{Q}}$ with $(n_a, n_p, n_b) = (2,3,2)$, $P = [p_1, p_2, p_3]$, $B = [b_1, b_2]$ and $A = [a_1, a_2]$. Two vertices b_1 and b_2 are introduced and the function $g : \mathbb{Z}_{n_a \times n_p} \to \mathbb{Z}_{n_b}$ is defined as $g(1,1) = 1$ and $g(2,3) = 2$.

Definition 3.3. *Consider a matrix $\mathbf{G} \in \{-1,0,1\}^{n_a \times n_p}$ and use function $g(h,j) = k$ to define index $k = 1, \ldots, n_b$ for the elements $\mathbf{G}_{hj} = -1$. The circuit $E_{\mathbf{G}}$ on $(n_a + n_p + n_b)$ qubits applies controlled-Z operations between qubits p_j and a_h if $\mathbf{G}_{hj} = 1$ and, between qubits $b_{g(h,j)}$ and a_h, and, $b_{g(h,j)}$ and p_j, when $\mathbf{G}_{hj} = -1$.*

Now we reformulate a lemma from [16] in order to translate bridge and break operations from graph theoretical ideas into operations on quantum states.

Lemma 3.1. *Consider a quantum state $E_{\mathbf{Q}} |\phi\rangle$ where $|\phi\rangle$ is arbitrary. If $\widetilde{\mathbf{Q}}$ is an extended IQP graph built from $\mathbf{Q}$ then there exists a state $E_{\widetilde{\mathbf{Q}}} |\psi\rangle$, which can be transformed into the state $E_{\mathbf{Q}} |\phi\rangle$ through a sequence of Pauli-Y basis measurements on qubits and local rotations around the Z axis on the unmeasured qubits through angles $\left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}$.*

The detailed proof of Lemma 3.1 shows us that we can create the following state where p_j and a_h are the primary and ancillary qubits connected to b_k.

$$\prod_{k=1}^{n_b} \left(S_{p_j}^{(-1)^{s_k^b + r_k^b}} \otimes S_{a_h}^{(-1)^{s_k^b + r_k^b}} \right)^{d_k^b} \left(Z_{p_j}^{r_k^b} \otimes Z_{a_j}^{r_k^b} \right)^{1-d_k} E_{\mathbf{Q}} |\phi\rangle \tag{3}$$

To achieve this, measurements of the qubits corresponding to bridge and break vertices (which we call *bridge and break qubits*) of $E_{\widetilde{\mathbf{Q}}} |\psi\rangle$ in the Pauli Y basis are made. The quantity s_k^b is the outcome of this measurement on qubit b_k, where said qubit was initialised in the state $|b_k\rangle = Y^{r_k^b} \sqrt{Y}^{d_k^b} |0\rangle$. Although using this method we could only generate $E_{\mathbf{Q}} \bigotimes_1^{n_a+n_p} |+\rangle$ (the state built in Lemma 2.1) up to some S corrections, these may be accounted for by correcting the primary and ancillary measurement bases. Hence, it is possible to perform an IQP computation by producing the required state in this way.

3.2 Blindness

To address problem 3, we would wish to construct the *Ideal Resource* of Figure 5 which takes as input from the Client an IQP computation, $(\mathbf{Q}, \theta)$, and in return gives a classical output $\widetilde{x}$. If the Server is honest, $\widetilde{x}$ comes from the distribution corresponding to $(\mathbf{Q}, \theta)$. If the Server is dishonest, they can input some quantum operation $\mathscr{E}$ and some quantum state ρ_B and force the output to the Client into the classical state $\mathscr{E}(\mathbf{Q}, \theta, \rho_B)$. We would like for the Server only to receive a extended IQP graph $\widetilde{\mathbf{Q}}$ which can be built from $\mathbf{Q}$, the distribution $\mathscr{Q}$ over the possible $\mathbf{Q}$ from which $\widetilde{\mathbf{Q}}$ could be built, and θ. Let us assume that this is public knowledge.

Blindness is added to the work of Section 3.1 by performing random rotations when initialising the primary and ancillary qubits. These are corrected by rotating the measurement bases of those qubits; ensuring the original IQP computation is performed. Intuitively, this randomness provides blindness as

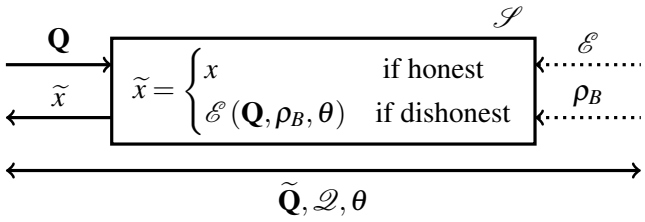


Figure 5: The ideal blind delegated IQP computation resource.

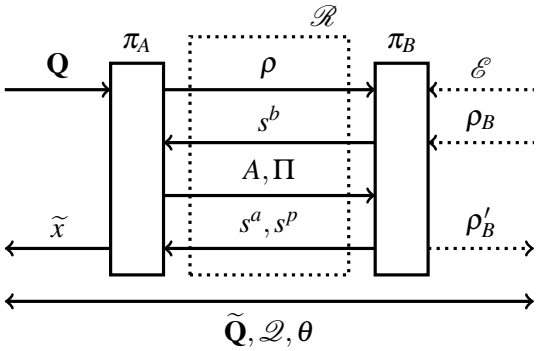


Figure 6: The real communication protocol. π_A is the set of operation performed by the Client, π_B are those of the Server and $\mathscr{R}$ is the communication channel (quantum and classical) used by the Client and the Server in the protocol.

it hides the previous corrections shown in equation (3), which would otherwise give away if a bridge or break operation was applied to a neighbour. Our real protocol can be seen in Algorithm 1 and Figure 6.

To prove composable security of the proposed protocol we drop the notion of a malicious Server for that of a global distinguisher which has a view of all inputs and outputs of the relevant resources. To recreate the view of a malicious Server, we develop a simulator σ interfacing between the ideal resource $\mathscr{S}$ of Figure 5 and the distinguisher in such a way that the latter cannot tell the difference between an interaction with the ideal resource interfacing with the simulator and the real protocol. We employ the Abstract Cryptography framework introduced in [26, 32] and teleportation techniques inspired by [13] to prove security in the case of a malicious Server. We have proven:

$$\pi_A \mathscr{R} \equiv \mathscr{S} \sigma \tag{10}$$

Theorem 3.1. *The communication protocol described by Algorithm 1 is information theoretically secure against a dishonest Server.*

We can now be sure that our communication protocol is indistinguishable from an ideal resource which performs an IQP computation without communicating any information to the Server which is not already public. This is proven in a composable framework [32, 13] and so can be used as part of future protocols, as we will do in section 4.

Algorithm 1 Blind distributed IQP computation

Public: $\widetilde{\mathbf{Q}}, \mathscr{Q}, \theta$
Client input: $\mathbf{Q}$
Client output: $\widetilde{x}$
Protocol:

1: The Client randomly generates $r^p, d^p \in \{0,1\}^{n_p}$ and $r^a, d^a \in \{0,1\}^{n_a}$ where n_p and n_a are the numbers of primary and ancillary qubits respectively.
2: The Client generates the states $\left|p_j\right\rangle = Z^{r_j^p} S^{d_j^p} \left|+\right\rangle$ and $\left|a_h\right\rangle = Z^{r_h^a} S^{d_h^a} \left|+\right\rangle$ for $j \in \{1,\ldots,n_p\}$ and $i \in \{1,\ldots,n_a\}$
3: Client creates $d^b \in \{0,1\}^{n_b}$ in the following way: For $h = 1,\ldots,n_a$ and $j = 1,\ldots,n_p$, if $\widetilde{\mathbf{Q}}_{hj} = -1$ and $\mathbf{Q}_{hj} = 0$, then $d_k^b = 0$ else if $\widetilde{\mathbf{Q}}_{hj} = -1$ and $\mathbf{Q}_{hj} = 1$ then $d_k^b = 1$. He keeps track of the relation between k and (h, j) via the surjective function $g : \mathbb{Z}_{n_a \times n_p} \to \mathbb{Z}_{n_b}$.
4: The Client generates $r^b \in \{0,1\}^{n_b}$ at random and produces the states $\left|b_k\right\rangle = Y^{r_k^b} \left(\sqrt{Y}\right)^{d_k^b} \left|0\right\rangle$ for $k \in \{1,\ldots,n_b\}$
5: State ρ comprising of all of the Client's produced states is sent to the Server.
6: The Server implements $E_{\widetilde{\mathbf{Q}}}$.
7: The Server measures qubits $b_1,\ldots,b_{n_b}$ in the Y-basis $\left\{\left|+^Y\right\rangle, \left|-^Y\right\rangle\right\}$ and sends the outcome $s^b \in \{0,1\}^{n_b}$ to the Client.
8: The Client calculates $\Pi^z, \Pi^s \in \{0,1\}^{n_p}$ and $A^z, A^s \in \{0,1\}^{n_a}$ using equations (4), (5), (6) and (7).

$$\Pi_j^z = \sum_{h,k:g(h,j)=k} r_k^b \left(1 - d_k^b\right) - r_j^p \tag{4}$$

$$\Pi_j^s = \sum_{h,k:g(h,j)=k} (-1)^{s_k^b + r_k^b} d_k^b - d_j^p \tag{5}$$

$$A_h^z = \sum_{j,k:g(h,j)=k} r_k^b \left(1 - d_k^b\right) - r_h^a \tag{6}$$

$$A_h^s = \sum_{j,k:g(h,j)=k} (-1)^{s_k^b + r_k^b} d_k^b - d_h^a \tag{7}$$

9: The Client sends $A \in \{0,1,2,3\}^{n_a}$ and $\Pi \in \{0,1,2,3\}^{n_p}$ for the ancillary and primary qubits respectively, where $A_h = A_h^s + 2A_h^z \pmod 4$ and $\Pi_j = \Pi_j^s + 2\Pi_j^z \pmod 4$.
10: The Server measures the respective qubits in the basis below for the ancillary and primary qubits respectively.

$$S^{-A_h} \left\{\left|0_\theta\right\rangle, \left|1_\theta\right\rangle\right\} \text{ and } S^{-\Pi_j} \left\{\left|+\right\rangle, \left|-\right\rangle\right\} \tag{8}$$

The measurement outcomes $s^p \in \{0,1\}^{n_p}$ and $s^a \in \{0,1\}^{n_a}$ are sent to the Client.
11: The Client generates and outputs $\widetilde{x} \in \{0,1\}^{n_p}$ as follows.

$$\widetilde{x}_j = s_j^p + \sum_{h:\mathbf{Q}_{hj}=1} s_h^a \pmod 2 \tag{9}$$

4 The Hypothesis Test

4.1 Previous work

The general idea of our *Hypothesis Test*, building on the work of [35], is that there is some hidden structure in the program elements, $\mathbf{q}_h$, of the X-program, which results in some structure in the distribution of the outputs, known only to the Client. The Client can use this structure to check the Server's reply. A Server possessing an IQP machine can reproduce this structure by implementing the X-program. A Server not in possession of an IQP machine cannot generate outputs obeying the same rules. We summarise this discussion by three conditions that a hypothesis test using this method must meet.

1.1 A Client asks a Server to perform a hard to classically simulate IQP computation.

1.2 The Client can check the solution to this computation because they know some secret structure that makes this checking processes efficient.

1.3 The Server must be unable to uncover this structure in polynomial time.

The particular 'known structure' of the output used in [35] to satisfy 1.2 is its *bias*.

Definition 4.1. *If X is a random variable taking values in $\{0,1\}^{n_p}$ and $\mathbf{s} \in \{0,1\}^{n_p}$ then the bias of X in the direction $\mathbf{s}$ is $\mathbb{P}\left(X \cdot \mathbf{s}^T = 0\right)$ where the product is performed modulo 2. Hence, the bias of a distribution in the direction $\mathbf{s}$ is the probability of a sample from the distribution being orthogonal to $\mathbf{s}$.*

To calculate the bias of X in direction $\mathbf{s} \in \{0,1\}^n$, we form the linear code $\mathscr{C}_\mathbf{s}$ by selecting all rows, $\mathbf{q}_h$, of the X-program, $(\mathbf{Q}, \theta) \in \{0,1\}^{n_a \times n_p} \times [0, 2\pi]$, such that $\mathbf{q}_h \cdot \mathbf{s}^\mathbf{T} = \mathbf{1}$. We form, from them, the matrix, $\mathbf{Q}_\mathbf{s}$, set as the generator matrix of $\mathscr{C}_\mathbf{s}$. Defining $n_\mathbf{s}$ as the number of rows of $\mathbf{Q}_\mathbf{s}$ allows us to understand the following expression derived in [35].

$$\mathbb{P}\left(X \cdot \mathbf{s}^T = 0\right) = \mathbb{E}_{\mathbf{c} \sim \mathscr{C}_\mathbf{s}} \left[\cos^2\left(\theta\left(n_s - 2 \cdot \#\mathbf{c}\right)\right)\right] \tag{11}$$

Hence, the bias of an X-program in the direction $\mathbf{s}$ depends only on θ and the linear code defined by the generator matrix $\mathbf{Q}_\mathbf{s}$. One can now imagine a hypothesis test derived from this. Although the X-program to be implemented needs to be made public, the direction $\mathbf{s}$ which will be used for checking, will be kept secret. This gives a Client, with the computational power to calculate the quantity of expression (11), the necessary information to compute the bias, but does not afford the Server the same privilege.

What we want to show is that the only way for the Server to produce an output with the correct bias is to use an IQP machine. If the Server could uncover $\mathbf{s}$ then they could calculate the value of expression (11) and return vectors to the Client which are orthogonal to $\mathbf{s}$ with the correct probability. We specialise the conditions mentioned at the beginning of this section to this particular method.

2.1 The X-Program sent to a Server represents an IQP computation that is hard to classically simulate.

2.2 It must be possible for a Client, having knowledge of a secret $\mathbf{s}$ and the X-program, to calculate the quantity of expression (11).

2.3 The knowledge of the Server must be insufficient to learn the value of $\mathbf{s}$.

In [35] the authors develop a protocol for building an X-program and a vector $\mathbf{s}$ performing this type of hypothesis test. The code $\mathscr{C}_\mathbf{s}$ used is a quadratic residue code with $\theta = \frac{\pi}{8}$ and condition 2.1 is conjectured to be satisfied by X-program matrices generating this code space. This conjecture is supported by giving a classical simulation that is believed to be optimal and achieves maximum bias value 0.75; different from that expected from an IQP machine. Condition 2.2 is satisfied by the construction

in [35], by proving that the bias value, which is $\cos^2\left(\frac{\pi}{8}\right)$ for their choice of X-program and $\mathbf{s}$, can be calculated in polynomial time.

The way in which condition 2.3 is addressed in [35] relies on the fact that the right-hand side of equation (11) is equal for all generator matrices in a *matroid* [31].

Definition 4.2. *A h-point binary* matroid *is an equivalence class of matrices with h rows, defined over* $\mathbb{F}_2$*. Two matrices,* $\mathbf{M}_1$ *and* $\mathbf{M}_2$*, are said to be equivalent if, for some permutation matrix* $\mathbf{R}$*, the column echelon reduced form of* $\mathbf{M}_1$ *is the same as the column echelon reduced form of* $\mathbf{R}\cdot\mathbf{M}_2$ *(In the case where the column dimensions do not match, we define equivalence by deleting columns containing only* 0*s after column echelon reduction).*

In order to move to a new matrix within the same matroid, consider the right-multiplication with matrix $\mathbf{A}$ on $\mathbf{Q}$. Notice that $\mathbf{q}_h\mathbf{s}^T = (\mathbf{q}_h\mathbf{A})\left(\mathbf{A}^{-1}\mathbf{s}^T\right)$. Rows which were originally non-orthogonal to $\mathbf{s}$ are now non-orthogonal to $\mathbf{A}^{-1}\mathbf{s}^T$, hence we can locate $\mathbf{Q}_\mathbf{s}$ in $\mathbf{Q}$ by using $\mathbf{A}^{-1}\mathbf{s}^T$. A way to hide $\mathbf{s}$ is therefore to randomise it with such an operation $\mathbf{A}$. We now understand what to do to the X-program we are considering, so that the value of the bias does not change. To increase the hiding of $\mathbf{s}$, the matrix might also include additional rows orthogonal to $\mathbf{s}$, which do not affect the value of the bias. The combination of matrix randomisation and the addition of new rows makes it hard, as conjectured in [35], up to some computational complexity assumptions, for the Server to recover $\mathbf{s}$ from the matrix that it receives. It is now simply a matter for the Server to implement the X-program and for the Client to check the bias of the output in the direction $\mathbf{s}$. This is the approach used by [35] to address condition 2.3.

4.2 Our Protocol

The main contribution of this work is to revisit condition 2.3.

Theorem 4.1. *Algorithm 2 presents an information-theoretically secure solution to condition 2.3.*

In Algorithm 2 we provide a hypothesis test that uses Algorithm 1 to verify quantum superiority. By using the blind IQP computation resource of Section 3.2 we have solved condition 2.3 but do so now with information theoretic security as opposed to the reliance on computational complexity assumptions used by [35]. This is true because the Server learns only the distribution $\mathscr{Q}$ over the possible set of graphs $\mathbf{Q}$. By setting $\mathbf{Q} = \mathbf{Q}_\mathbf{s}\mathbf{A}$, Algorithm 2 develops a bijection mapping $\widehat{\mathbf{s}} \in \{0,1\}^{n_p-1}$ to a unique matrix $\mathbf{Q} \in \{0,1\}^{n_a\times n_p}$. So $\mathscr{Q}$ is equivalent to the distribution from which $\widehat{\mathbf{s}}$ is drawn. In this case it is the uniform distribution over a set of size 2^{n_p-1}.

5 Discussion, Conclusion and Future Work

We have presented a new certification technique for IQP machines which can be run by a client able to prepare single-qubit Pauli eigenstates. By giving the Client minimal quantum abilities we can remove computational restrictions placed on the Server in previous work [35] and, instead, prove information-theoretical security against an untrusted Server.

There are several advantages of using this tailored verification protocol for IQP computations, rather than a straightforward verification in a universal quantum computing model [16, 24]. The latter requires higher precision in the manipulation of single qubits from the client (use of states other than single-qubit Pauli eigenstates) and significantly more quantum communication and processing for the verification technique. Although the qubit consumption there is still, as in this work, linear in the size of the computation, in early machines the constant factor will likely be important. Further, asking a Server to perform

Algorithm 2 Our hypothesis test protocol

Input: n_a prime such that n_a+1 is a multiple of 8.
Client output: $o \in \{0,1\}$
Protocol:

1: Set $n_p = \frac{n_a+1}{2}$
2: Take the quadratic residue code generator matrix $\mathbf{Q_r} \in \{0,1\}^{n_a \times (n_p-1)}$
3: Let $\mathbf{Q_s} \in \{0,1\}^{n_a \times n_p}$ be $\mathbf{Q_r}$ with a column of ones appended to the last column.
4: Pick $\widehat{\mathbf{s}} \in \{0,1\}^{n_p-1}$ chosen uniformly at random.
5: Define the matrix $\mathbf{A} \in \{0,1\}^{n_p \times n_p}$ according to equation (12).

$$\mathbf{A}_{h,j} = \begin{cases} 1 & \text{if } h = j \\ 0 & \text{if } h \neq j \text{ and } j < n_p \\ \widehat{\mathbf{s}}_h & \text{if } j = n_p \text{ and } h < n_p \end{cases} \tag{12}$$

6: Set $\mathbf{Q} = \mathbf{Q_s}\mathbf{A}$ and $\theta = \frac{\pi}{8}$.
7: Set $\widetilde{\mathbf{Q}}$ to be the matrix $\mathbf{Q_r}$ with a column of -1 appended.
8: Set $\mathscr{Q}$ to be the uniform distribution over all possible $\mathbf{Q}$ for different $\widehat{\mathbf{s}}$.
9: Perform the IQP computation $\mathbf{Q}$ using Algorithm 1 with inputs $\mathbf{Q}$, $\widetilde{\mathbf{Q}}$, $\mathscr{Q}$ and θ and outputs $\widetilde{x}$ and ρ'_B.
10: Let $\mathbf{s} \in \{0,1\}^{n_p}$ be the vector with entries all equal to zero with the exception of the last which is set to one.
11: Test the orthogonality of the output $\widetilde{x}$ against $A^{-1}\mathbf{s}^T$ setting $o = 0$ if it is not orthogonal and $o = 1$ if it is orthogonal.

an IQP computation using a model that is universal for quantum computation would require the Server to create large cluster states and perform measurement that might lie far beyond its IQP capabilities.

IQP circuits are important as they may prove easier to implement experimentally compared to universal quantum computers. Due to the commutativity of the gates it is theoretically possible to perform an IQP computation in one round of measurements. Our protocol requires a two round MBQC computation which we believe not to be a significant additional requirement and which provides an important improvement compared to the implementation requirements of universal quantum computations. This could make our scheme implementable, for a small number of qubits, in the near term, and so a future avenue of research would be to study this hypothesis test protocol under realistic experimental errors, following similar examples of work in this direction [8, 23].

The demand for the Server to have memory to support a two round MBQC computation means machines capable of passing the original test of [35] might not be able to pass that of this work. There they do not restrict the architectures that the Server can use, which comes at the high cost of placing computational restrictions on the Server [35]. Our requirement that the Server can perform two round MBQC allows us to achieve information-theoretic security.

Finally, given that Gaussian quantum subtheory can be seen as a continuous variable analogue of the stabiliser formalism [37, 4, 3], a natural extention would be a continuous variable analogue of our protocol where the Client prepares only Gaussian states.

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